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FIXED EXCHANGE RATES AND NON-COOPERATIVE MONETARY POLICIES

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INTRODUCTION

It is widely agreed that a fixed exchange rate system does not in some sense work unless countries somehow surrender their freedom to finance expenditures by money creation[1] --in other words, unless they somehow coordinate[2] their money creation policies. There are, however, several possible meanings of a fixed rate system "not working" absent coordination.[3] One is that no equilibrium exists. Another is that the fixed exchange rate system is abandoned. In this paper, in the context of a simple two-country model, I study the equlibria of several games in which countries choose money creation policies. I show explicitly that absent coordination fixed exchange rate systems do not work in the sense that either the equilibrium is undesirable in a welfare sense, or an equilibrium of the game involves abandoning the fixed rate system, or both.

The model use is a two-country, pure exchange overlapping generations model. Other intertemporal equilibrium[4] models with no rate of return dominance across currencies display an incentive for "international seignorage". However, the simple versions of cash-in-advance models (see Lucas (7), Helpman (10), Sargent (25) and Svensson (27)) do not provide any welfare loss associated with inflation. They, thus, imply unlimited use of an inflation tax in a fixed exchange system. In the OG model, since money is used as a store of value, there is a non-trivial choice between the international

inflation tax and the distortion created at home. Given my purposes, the existence of this non-trivial choice and the simplicity of the model are its main virtues.

The model draws heavily on the framework used by Kareken and Wallace (14) to demonstrate the indeterminacy of flexible exchange rates if there are no restrictions on portfolio choice. A strategic approach in which countries' choices are modelled as a game is used.[5]

The paper proceeds as follows. The environment (demographics, preferences, endowments and informational structure) is described in Section I. In Section II we present the competitive mechanism in which private agents solve their optimization problems given the time paths of government actions and in which markets are required to clear. Section III introduces the "international seignorage" incentive providing the case of a government facing the non-trivial choice of an optimal combination of monetary and fiscal policies in a world without capital controls and with the other government passive. This is contrasted with the comparable choice for a closed economy. In Section IV the situations in which governments react to each other's choice is modelled in a non-cooperative game in which each government attempts to maximize the steady state utility of its own representative agent. Three games are presented, the games differing as to the strategies available to the governments. In a first game, both countries are constrained to choose once-and-for-all real seignorage choices consistent with the maintenance of a fixed exchange rate stationary monetary equilibrium with perfect currency substitution. In a second game, governments may abandon the system by choosing policies which imply a non-monetary equilibrium with barter international borrowing and lending. In a third game, governments may abandon the system by choosing either the barter regime or by choosing an autarkic portfolio regime in which a country has its own money and is completely insulated from the monetary policy of the other country, a version of a flexible exchange rate system. The Nash Equilibria of these games, in which each player takes its rival's actions as given, are analyzed.

The main results are the following. For the first two games the Nash equilibria yield non-optimal allocations, either because an inflationary monetary equilibrium prevails or because countries abandon the system for the non-monetary equilibrium. This leaves room for international coordination. It is shown that there always exist a combination of monetary choices that provide Pareto Superior allocations to the Nash outcome. In the third game, when the portfolio autarky option is introduced as a strategy choice, it becomes the Nash Equilibrium of the game. In some cases such an outcome is optimal.

In summary, this paper presents a model that reproduces both the necessity for international coordination for a fixed rate system to work, as well as reporducing the fact of

foreign exchange controls being imposed as a way to insulate economies from external inefficiencies.

SECTION I. THE MODEL

There are two countries ($h=i,j$) consisting of two-period lived overlapping generations. There is only one perfect perishable good in the economy. The environment is stationary, and represented by the following:

* The population is constant, $N^h(t) = N^h$
* Endowments: $(\omega_{1h}(t), \omega_{2h}(t)) = (\omega_{1h}, \omega_{2h})$, $h=i,j$

* Preferences are identical: $U^h(c_{1h}(t), c_{2h}(t)$ = $U(c_{1h}, c_{2h})$, and satisfy the standard assumptions: U is continuous, twice differentiable, homothetic, concave. Let $v(c_{1h}/c_{2h}) = U_1/U_2$. Then it is assumed that (i) $v > 0$, $v' < 0$ (ii) $(c_{1h}/c_{2h}) \cdot v'/v \geq -1$, and $U_2 \to +\infty$ monotonically as $c_{2h} \to 0$. For most of what I do I assume that Cobb-Douglas preferences given by $U(c_{1h}, c_{2h}) = c_{1h} \cdot c_{2h}$.

  These preferences satisfy the above assumptions as well as others used in related literature and that we shall use later on.
* There are no production or storage technologies available.
* Finally, there are two infinitely lived governments, one in each country. The economy lasts forever. All the agents have perfect foresight.

Notice that there is no physical separation, or preferences or technological differentiation that can define a "country". A "country" here is just a subset of agents that can be taxed by only one of the governments and for whom only that government cares.

In particular, the government will care about the welfare of a representative generation in the history of this economy. In a stationary environment this is done by maximizing the steady state utility. This implies that the government does not take into account the welfare of the old at time t=1, but of the rest of the generations from then on.

Each government consume $G^h(t)$ units of time t good, whose purchase has to be financed either by raising lump-sum taxes or through seignorage.

Each government has the monopoly power of printing its country's currency. At period t, the money creation in country h will be, in real terms:

$$\frac{M^h(t) - M^h(t-1)}{P_h(t)}, \quad h=i,j$$

where $M^h(t)$ is the money supply of country h at time t, of pieces of paper that can be exchanged for $1/P_h(t)$ units of time t good in country h (i.e., $P_h(t)$ is the price level of country h at time t).

At each period of time, we can define the exchange rate to be

$$e(t) = \frac{P_i(t)}{P_j(t)} \tag{1}$$

Besides money creation, each government can impose lump-sum taxes or give lump-sum monetary transfers. In real terms $\tau^h(t) = N^h\beta_h(t)w_{1h}(t)$ where β_h is per unit tax (if $\beta_h<0$, it will be a transfer). We are assuming here that taxes are imposed in the first period's endowments.

The government budget constraint will be, therefore, given by

$$(2)\quad \frac{M^h(t) - M^h(t-1)}{P^h(t)} + \tau^h(t) = G^h(t) \text{ all } t \geq 1$$

or

$$\frac{M^h(t) - M^h(t-1)}{P_h(t)} = G(t) - N^h\beta_h(t)w_{1h}(t)$$

Initially there are $M^h(0)$ units of fiat money of country h held in an indeterminate composition by the old at time t-1. Without loss of generality, we can assume that the initial old hold the original notes of their country of origin, that is, $M^h(0) = N^h \cdot \bar{m}^h(0)$, $h=i,j$, where $\bar{m}^h(0)$ is the stock of fiat (domestic) currency in the hands of a representative old at time t=1, country h.

The government, besides using monetary/fiscal policies, has yet another way to affect the behavior (and the welfare) of the private agents. It can change the <u>regime</u> under which private trading occurs, by imposing portfolio restrictions on holding foreign stock of currency, as well as trade autarky (this will be called the "portfolio autarky" regime), or by not allowing valued money to exist, switching to a barter economy (or "non-monetary" regime).

We can define "trade balance" for country h to be, at time h, for the case where there are no restrictions:

$$(3)\quad TB_h(t) = w_{1h}(t) + w_{2h}(t) - c^h_t - c^h_{t+1}$$

As we'll see later, this "trade balance" does not reflect movements in the current account, but rather in the capital account, since it consists of international borrowing and lending.

<u>SECTION II. COMPETITIVE EQUILIBRIA FOR GIVEN POLICIES</u>

In this Section we analyze the behavior of the private sector, given the choice of policies of the governments.

For each possible choice of <u>regime</u>, we analyze the competitive equilibria associated with the <u>monetary</u> or <u>fiscal</u> policies that each government may use, and that the private agents take as given.

There are three groups of possible equilibria, one per each choice of regime:

<u>Monetary Equilibria:</u> a regime in which fiat monies are valued and there are no restrictions on portfolio holding, or on international borrowing and lending. Under this regime, there is trade between generations, and between members of different countries in the same generation. This is the equivalent to a "laissez faire" regime, but in which each government can affect the private sector by the way of monetary and fiscal policies, represented on the choice of sequen-

ces $G_h(t)$, $\beta_h(t)$, which will pin down the choice of money creation to satisfy the government budget constraint.

<u>Non-monetary Equilibria</u>: a regime in which, by government choice, fiat money has no value. As we will see later this could be implemented through the announcement of a monetary policy incompatible with existence of a competitive monetary equilibrium. Under this regime there is trade between members of the same generation in different countries (international borrowing and lending in a barter economy) but no trade between generations. The government can still affect the economy through real taxation, but if $G_h(t) = 0$, it will never do so.

<u>Portfolio Autarky Equilibria</u>: a regime under which each country goes on its own, that is, foreign money cannot be held domestically and there is no borrowing and lending across the border. We will discuss later why these two restrictions must go together. Within this regime, the government can use closed economy monetary and fiscal policies and/or switch to a non-monetary regime. We will see that once this portfolio autarky is chosen, the policy choice will always be a monetary non-distortionary one when $G_h(t) = 0$.

<u>II.1. MONETARY EQUILIBRIA</u>

<u>Consumer's problem</u>: Given the relevant prices $((P_h(t), P_h(t+1))_{h=1,j}, r(t))$ the endowments $(w_{th}, w_{t+1,h})$ and the tax (transfer) sequence $\beta_h(t)$, each representative consumer of

country h, generation $t \geq 1$. will choose a vector of consumption $c^h = (c^h_t, c^h t+1)$, a stock of fiat money $(m^h_k(t))_{k=1,j}$ and engage in loans $l^h(t)$ in order to:

$$(4)\quad \max U(c^h_t, c^h_{t+1})$$

$$\text{s.t. } c^h_t + \sum_{k=1,j} (m^h_k(t)/P_k(t)) + l^h(t) \leq \overline{w}_{th}(1-\beta_h(t))$$

$$c^h_t+1 \leq w_{t+1,h} + \sum_{k=1,j} (m^h_k(t)/P_k(t+1)) + l^h(t)(1+r(t))$$

$$c^h_t, c^h_{t+1}, m^h_k(t) \geq 0$$

where $m^h_k(t)/P_k(t)$ is the demand for money k (in terms of time t good) by consumer h, and $1+r(t)$ is the real return on the (internationally unrestricted) private loans market.

Notice that, because of the two-period overlapping generations structure and because of all agents being homogenous within a country, all loans will take place between members of the <u>same</u> generation of different countries. And, because of there only being one good, all trade imbalances will be of the capital account type (i.e., international borrowing and lending).

Finally, we can consider the optimization problem of the members of generation 0, old at t+1, which is max $U(c^h_2(0))$ by the choice of $c^h_2(0)$ subject to $c^h_2(0) \leq w_{2h} + (m^h(0)/P_h(1))$.

<u>Definition 1</u>: Competitive Monetary Equilibrium with both monies being valued and no restrictions (ME).

For _each_ pair of sequences $\beta_i(t)$ $\beta_j(t)$, a ME is a set of sequences $P_i(t)$ $P_j(t)$ $r(t)$ $e(t)$ and of allocations such that:

(i) Each consumer of generation $t \geq 1$ solves their optimization problem (4), taking as given the relevant prices, exchange rate and policy parameters.

(ii) Both monies are valued i.e., $1/P_k(t) > 0$ all k,t.

(iii) The old at time t=1 solve their (trivial) optimization problem.

(iv) Markets clear period by period, for both currencies, namely

$$\Sigma_h m_k^h(t)/P_k(t) + \Sigma_h l^h(t) = M_k(t)/P_k(t),$$
$$k=i,j, \text{ all } t\geq 1.$$

Since there are no legal restrictions on portfolio choice, (ii) implies that both currencies must _yield the same real return_ (dominance result), which must equal the (real) interest rate on international loans, i.e.,

(5) $$\frac{P_i(t+1)}{P_i(t)} = \frac{P_j(t+1)}{P_j(t)} = \frac{1}{1+r(t)}$$

Note that this condition, together with the definition of the exchange rate (1), implies (as an equilibrium condition) _constancy_ of the exchange rate, i.e., $e(t+1) = e(t) = e$. This well-known result is a consequence of the absence of capital controls, and of having a single good.

Because of Walras Law (iv) implies that the good market is (world-wide) cleared, which obviously implies $TB^i(t) = -TB^j(t)$. (Note that, in general, we won't get a balanced capital account; that is, this model is able to generate a permanent surplus (deficit) capital account with fixed exchange rates, in an _equilibrium_ situation).

Indeterminancy of the flexible exchange rate: As shown in Kareken and Wallace (14), absence of capital controls make the (constant) exchange rate (if to be left set by market conditions) indeterminate in the following sense: for _each_ possible (unchanging) e, there is an (unrestricted) ME. Clearly the distribution of utility of the initial old depend heavily on which e might hold. But as we said before, the government will not take into account the welfare of these citizens. This indeterminacy could be solved either by "countries going on their own" (capital controls, also called portfolio autarky regimes) or by cooperative budget policies where the (constant) exchange rate is fixed at e. Nickelsburg (23) has also shown a case where the indeterminancy disappears under a flexible exchange rate: when there is a threat of the governments imposing a portfolio autarky regime at some (either known or random) date t+T.

In what follows I will assume that the exchange rate is fixed (by mutual agreement, or by unilateral action) at e.

Definition 2: A Stationary Competitive Monetary Equilibrium With Fixed Exchange Rates (MEF) is a ME such that given the exogenous process $\beta_h(t) = \beta_h$, all t, h=i,j.

$$1 + r(t) = 1+r \quad \text{all} \quad t \geq 1$$

$$(c^h_t(t), c^h_{t+1}(t)) = (c_{1h}, c_{2h}) \text{ all } t \geq 1$$

$$\frac{P_h(t+1)}{P_h(t)} = \frac{P_h(t)}{P_h(t-1)} \quad \text{all } t \geq 2$$

and where $e(t+1) = e(t) = e$ is fixed at some $e = \bar{e}$, all $t \geq 1$, either by agreement or by unilateral action of one of the countries.

Existence of such an equilibrium (see Kareken and Wallace (13) will allows us to concentrate on the stationary (steady state) solution.

In overlapping generation models there is usually a continuum of non-stationary monetary equilibria with limiting real balances equal to zero. However, we'll consider only those equilibria which are stationary, which will be well-defined given our stationary environment.

II.2. NON-MONETARY EQUILIBRIA

A non-monetary regime can be defined as an equilibrium situation where fiat monies are worthless, namely $1/P_h(t) = 0$ for some t. (Note that if $1/P_h(t+1) = 0$ for some h, then $1/P_h(t) = 0$ for that h, since there is perfect foresight).

We will allow barter trade to take place (as mentioned before this will take the form of international borrowing and lending between memebers of the same generation).

Here, since monies have no value, and there is only one good, the concept of exchange rate is meaningless. Trade will take place (in units of that good) against promises of payment for some units of the good in the next period).

Note that, in this case, the government must restrict $\beta_h \geq 0$, since there can't be monetary transfers from the government, but still taxes can be levied in the form of goods.

Consumer's problem: In the non-monetary regime, the consumers choose the best bundle that satisfies, given $\beta_h(t)$

(6) $$c^h_t + l_h(t) \leq w_{th}(1-\beta_h(t))$$

$$c^h_{t+1} \leq w_{t+1,h} + l^h(t)(1 + r_{NM}(t))$$

where $1 + r_{NM}(t)$ is the rate of return on the loans market in the non-monetary regime.

The optimization problem of the members of generation 0 will be, trivially, given by $c^h_2(0) = w_{2h}(0)$.

Finally, the government budget constraint is:

$N^h \beta_h(t) w_{1h} = G(t)$, $h=i,j$, all $t \geq 1$.

<u>Definition 4</u>: Non-Monetary equilibrium (NME)

For each pair of sequences $\{\beta_i(t), \beta_j(t)\}$, a NME is a sequence $r_{NM}(t)$ and of allocations such that:

(i) Each consumer of generation $t \geq 1$ solve their optimization problem (6), taking as given the policy parameters and the rate of return on loans.

(ii) Old at time $t=1$ solve their optimization problem.

(iii) International loans market clears period by period, namely, $\Sigma_h l^h(t) = 0$, all $t \geq 1$.

Absence of portfolio restrictions and existence of the international loans market make rates of return <u>equal</u> across countries, and therefore a monetary equilibrium in one country cannot coexist with a non-monetary equilibrium in the other.

<u>II.3. PORTFOLIO AUTARKY EQUILIBRIA</u>

We deal only with closed economy <u>monetary</u> equilibria. The closed economy non-monetary equilibria, given homogeneity of agents within borders, would mean each agent getting as consumption bundle their initial endowment vector, since they cannot trade with anybody. For the closed monetary economy:

<u>Consumer's problem</u>: will be identical to (4) where we impose $m^i_j(t) = 0$ for consumer i, and where each agent faces a country-specific $1+r_h(t)$. The same for the initial old.

<u>Definition 5</u>: A Competitive Monetary Equilibrium with Portfolio Autarky (MEPA) is a set of sequences $\{P_h(t), r_h(t)\}$ for each country h, and of allocations such that:

(i) Each consumer of generation $t \geq 1$, country h, solves their optimization problem mentioned above.

(ii) Old at time $t=1$ solve their trivial optimization problem.

(iii) Both monies are valued, $1/P_h(t) > 0$ all h,t.

(iv) Markets clear, period by period, for both currencies

$$M_h(t) = m^h{}_h(t) \quad \text{for } h=i,j, \text{ all } t \geq 1$$

From (iii)

$$(7) \qquad \frac{P_h(t+1)}{P_h(t)} = \frac{1}{1+r_h(t)}$$

The absence of the "dominance result," (5), implies that $e(t) = \dfrac{P_i(t)}{P_j(t)}$ will not necessarily be constant.

Note that the exchange rate does not play any role in the private agent's optimal choice, since both countries are completely isolated from each other. It is just an abstraction, the relative price of both monies, where there is no market available for such an exchange. Absence of such a market makes the concept of "flexible exchange rate," vacuous.

On the other hand, the governments do not need to set a "path of exchange rates," since their domestic monetary policies, chosen independently, will pin down $P_h(t)$, and therefore the ratio. Hence, the concept "fixed exchange rate" is also vacuous.

Definition 6: A Stationary Competitive Monetary Equilibrium with Portfolio Autarky, given $\beta_h(t) = \beta_h$ is a MEPA in which

$$1 + r_h(t) = 1 + r_h, \text{ all } h, t.$$

SECTION III. INCENTIVES TO INFLATE IN AN OPEN VERSUS A CLOSED ECONOMY: A PRELIMINARY DISCUSSION

We can easily identify the closed economy with the portfolio autarky regime and the open economy with the monetary regime without restrictions.

As mentioned before, each government cares only about the welfare of its own residents. (In this case, the utility level associated with the steady state allocation). On the other hand, the government can only impose taxes (or give monetary transfers) on those residents. In the closed economy, the government can only affect their own residents. In the open economy under fixed exchange rates and no capital controls, the government is able, in equilibrium, to affect the residents of the other country by way of printing money, and giving it as transfers to their residents, which may exchange it for goods at $P_h(t)$ (i.e., give it to foreigners who want it as a store of value) or keep it themselves as a store of value whose return is lower, in real terms, the higher the world-wide inflation. Under some conditions, basically on the distribution of endowments, the distortion created (lower return on savings) will be compensated by the gains of the transfers received. Therefore, if that were the case, it will pay for the government to do so. Symmetrically the government could tax and deflate (one might think of the government collecting taxes in money, and destroying it, or collecting it in the form of goods, and exchanging it for money to be destroyed). What matters here is how does the government affect the world money supply (world inflation) and the transfers/taxes imposed on their citizens. The key is that in the absence of capital controls and with fixed exchange rates, the (real) purchasing power of <u>additions to the money stock</u> may be positive. This opens the possibility of the government using money creation as an <u>optimal</u> way of financing its real deficit.

In the public finance literature, one of the main results is that lump-sum taxation, if costless, is best. Wallace (29) has shown a version of such a proposition, in the one country version of an economy like the one we are dealing with: any monetary equilibrium with a constant money supply and lump-sum taxes is Pareto Superior to any stationary equilibrium with the deficit being (totally or partly) financed through fiat money issue.

It turns out that when you consider a two-country version of such a model, and if <u>fixed</u> exchange rates prevail, there will be an incentive for that country's government to create some distortion, i.e., finance the deficit, at least partly, through money creation. We now present an example where the government will choose partly (or totally) to monetize depending on its country's relative (and on the deficit's relative) size.

<u>An example:</u>

Let $G_i(t)=G=\bar{\beta}_i w_{1i}>0$, $G_j(t)=0$, $N_h=1$, $U(.)=c_1{}^h.c_2{}^h$, $h=i,j$

and call money creation in i:

$$\frac{M^i(t) - M^i(t-1)}{P_i(t)} = \beta_i{}^M w_i i, \text{ all } t$$

taxes in i: $\tau_i(t) = \beta_i{}^T w_{1i}$, all t

then the government budget constraint implies, from (2),

$$G = \bar{\beta}_i w_{1i} = (\beta_i{}^M + \beta_i{}^T) w_{1i} \qquad \bar{\beta}_i = \beta_i{}^M + \beta_i{}^T.$$

The government has to choose the parameters $\beta_i{}^M$ (the proportion of expenditure financed through seignorage) and $\beta_i{}^T$ (the proportion of the expenditure raised by taxes).

For the above preferences, the consumer's optimization problem yield the following first order condtions:

$$(6) \quad \frac{c^h{}_t+1}{c_t{}^h} = 1+r \geq \frac{P_h(t+1)}{P_h(t)}$$

For a monetary equilibrium, (6) will hold with equality, and together with the budget constraints it yields the following individual h savings

$$\sum_{k=i,j} (\dot{m}^h(t)/P_k(t)) + l^h(t) = 1/2(w_{1h}(1-\beta_h{}^T))$$

From the market clearing condition for the above aggregate savings, summing over h,

$$1/2(w_{1i}(1-\beta_i{}^T) + w_{1j}) = \frac{M^i(t) + eM^j(t)}{P_i(t)}, \text{ all } t \geq 1$$

Using $M_j(t) = M_j(t-1)$, since $\beta_j^M = 0$, we can solve:

$$\frac{P_i(t)}{P_i(t+1)} = 1+r = \frac{w_{1i}(1-\beta_i^T)+w_{1j}-2w_{1i}\beta_i^M}{w_{1i}(1-\beta_i^T)+w_{1j}} = \frac{1-\alpha_i(2\beta_i^M+\beta_i^T)}{1-\alpha_i\beta_i^T},$$

where we call $\alpha_i = \frac{w_{1i}}{w_{1i}+w_{1j}}$, relative size of country i.

Note that if the country doesn't monetize at all (complete taxation)

$$\left.\begin{array}{l}\beta_i^M = 0\\ \beta_i^T = \bar{\beta}_i\end{array}\right\} \quad 1 + r = 1 \text{ (no distortion)}$$

On the contrary, if the government completely monetizes:

$$\left.\begin{array}{l}\beta_i^M = \bar{\beta}_i\\ \beta_i^T = 0\end{array}\right\} \quad 1+r = 1 - 2\alpha_i\bar{\beta}_i$$

Note that existence requires $1+r > 0$: we are going to assume that the environment and the size of the deficit allow for a monetary equilibrium to exist even in the case of complete monetization, i.e., $1-2\alpha_i\beta_i > 0$. (The deficit cannot be "too big").

<u>Choice problem</u> for government i: a combination of (β_i^M, β_i^T) such that the steady state utility of the representative agent of country i is maximized in the corresponding stationary competitive monetary equilibrium. Formally,

$$\max U_i(c_{1i}(\beta_i^T,\beta_i^M), c_{2i}(\beta_i^T,\beta_i^M)) = U_i(\beta_i^T,\beta_i^M)$$

$$\text{s.t.} \quad \beta_i^M + \beta_i^T = \bar{\beta}_i$$

$$\beta_i^M, \beta_i^M \geq 0.$$

It's important to emphasize that $\bar{\beta}_i$ is <u>not</u> a choice variable, (it it were, $\bar{\beta}_i = 0$ would always be the coice in this set-up), and that $\bar{\beta}_j = 0$ (the other country is passive).

For the specified utility function, given the conditions of a monetary equilibrium, the objective function will be:

$$U_i(\beta_i^T,\beta_i^M) = \frac{w^2_{1i}(1-\beta_i^T)^2}{4}\left(\frac{1-\alpha_i(2\beta_i^M + \beta_i^T)}{1-\alpha_i\beta_i^T}\right)$$

The first-order (Khun-Tucker) conditions yield the following system:

(a) For the interior solution $\beta_i^M, \beta_i^T > 0$

(i) $-2(1-\alpha_i(2\beta_i^M+\beta_i^T)) + \left(\frac{1-\alpha_i(2\beta_i^M+\beta_i^T)}{1-\alpha_i\beta_i^T}+3\right)\alpha_i(1-\beta_i^T)=0$

(ii) $\beta_i^M + \beta_i^T = \bar{\beta}_i$

b) For the corner solution, $\beta_i^M = \bar{\beta}_i$ or $\beta_i^T = \bar{\beta}_i$

The following features characterize the optimal choice for the government:

Let $(\beta_i^{M*}, \beta_i^{T*})$ represent the optimal policy package, given $\bar{\beta}_i$, and the physical environment (in this case, the relative size of the country i).

Then, the following can be easily shown:

(i) $\frac{\partial\beta_i^{M*}}{\partial\bar{\beta}_i} < 0$, $\frac{\partial\beta_i^{T*}}{\partial\bar{\beta}_i} > 0$, ceteris paribus, the bigger the deficit, the bigger proportion of taxation in the optimal package.

(ii) $\frac{\partial\beta_i^{M*}}{\partial\alpha_i} < 0$, $\frac{\partial\beta_i^{T*}}{\partial\alpha_i} > 0$, the bigger the country the smaller the optimal monetization.

(iii) β_i^{m*} 0 as α_i 1. (Wallace's case. Closed Economy)

$$\beta_i^{m*} \rightarrow \bar{\beta}_i \quad \text{as } \alpha_i \rightarrow 0.$$

We now present three numerical examples, each one represented graphically in Figures 1, 2, and 3, respectively. For those examples, $G = \bar{\beta}_i w_{1i}$, $\bar{\beta}_i = 0.2$, $w_{1i}=y=2$.

Example 1: (Wallace's case): one country $(\alpha_i=1)$, endowments (2,0).

	β_i^m	β_i^T	1+r	U_i	
Complete Monetization	0.2	0	0.60	0.60	
Complete Taxation	0	0.2	1	0.64	Optimal

Example 2: Two countries, identical $(\alpha_i=1/2)$, endowments (2,0).

	β_i^m	β_i^T	1+r	U_i	
Complete Monetization	0.2	0	0.80	0.80	Optimal
Complete Taxation	0	0.2	1	0.64	

Example 3: Country i much larger $(\alpha_i = 0.9)$, endowments (2,0)

	β_i^m	β_i^T	1+r	U_i	
Complete Monetization	0.2	0	0.64	0.64	
Complete Taxation	0	0.2	1	0.64	
Optimal Combination	0.1	0.1	0.80	0.65	Optimal

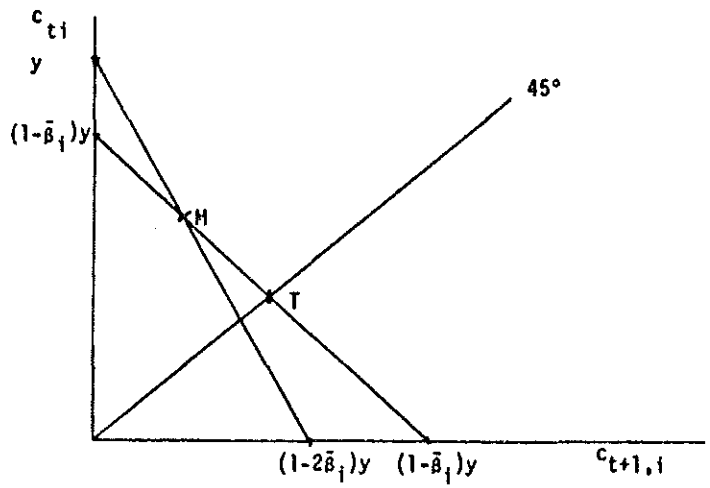
The examples are chosen such that the first two yield the corner solution as optimal policy, the last one a 50-50% combination. Clearly the welfare associated with a taxation scheme doesn't change from an open to a closed economy. "Fiscal" policies here are not transmitted (since they are non-distortionay). Only monetary policies are negatively transmitted. In summary, for the cases described in these examples, endowments in the second period are zero in both countries, (i.e., both representative consumers have a strong desire for saving (or "high tolerance for inflation". if you wish)). Then, what matters in this case is the relative size of the country and the relative size of the deficit.

The bigger the country (the deficit), the bigger the proportion of taxes in the optimal policy package. Wallace's case would be the limit of this case, when the country is so big that it's the only one (closed economy).

The case where portofolio restrictions are imposed (capital controls) and flexible exchange rates hold, is equivalent to the closed economy: each country suffers from its own inflation, and is isolated from the others, since the exchange rate will reflect any deviation from the growth of the total world money supply. Therefore the portfolio autarky-flexible exchange rate case is just the closed economy case where zero inflation (complete taxation) is the optimal policy.

In the next Section we consider the case of the <u>other</u> government, instead of being passive (in the above example it was actually non-existant) interacts by choosing the best policy given the action of its rival, and each one is looking forward to the welfare level of their own citizens.

<u>Figure 1</u>

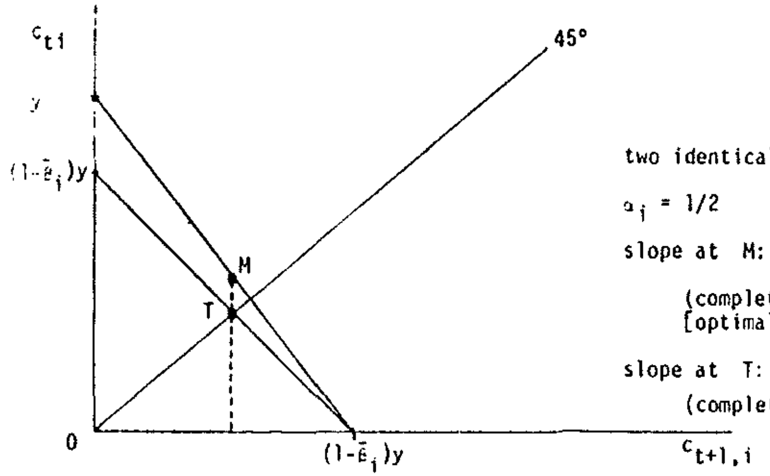


one country $(y,0)$, $\alpha_i = 1$

slope at M: $-\frac{1}{1-2\bar{\beta}_i}$ (Monetization)

slope at T: -1 (Taxation)

<u>Figure 2</u>



two identical countries $(y,0)$

$\alpha_i = 1/2$

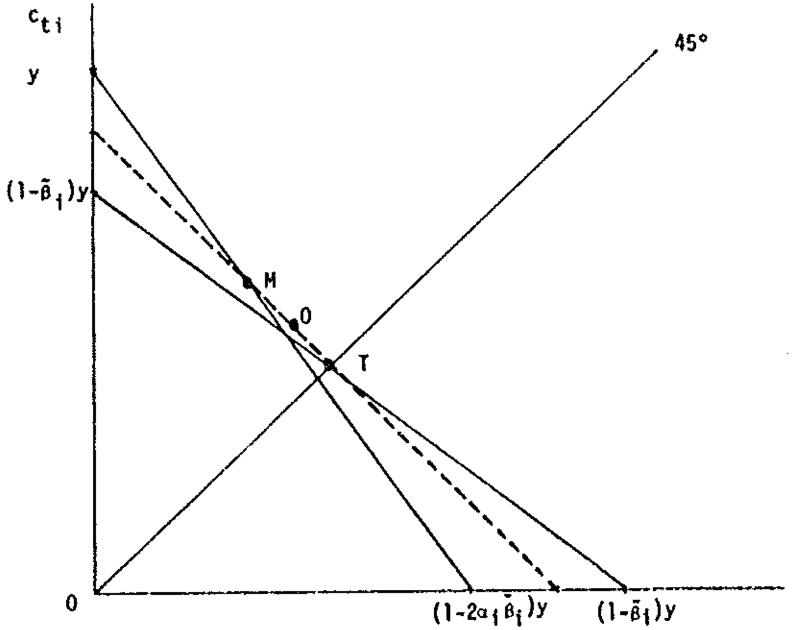
slope at M: $\frac{1}{1+r} = \frac{-1}{1-\beta_i}$

(complete monetization)
[optimal policy]

slope at T: $\frac{1}{1+r} = -1$

(complete taxation)

Figure 3



two countries $(y,0)(0.1y,0)$, $\alpha_i = 0.9$ (country i very big)

M: complete monetization

T: complete taxation

O: optimal policy

SECTION IV. NASH EQUILIBRIA

IV.1. INTRODUCTION

Given the incentive to use monetary policies in an open economy with a fixed exchange regime, it is immediate to establish a non-cooperative game in which both players (the governments of each country) engage in choosing from a set of regimes and policy parameters (strategy spaces), and whose outcome (payoff function) is going to be determined by the combined, simultaneous actions of the players.

In this Section we consider the Nash equilibrium concept, in which each player takes its rival action as given, and where a situation is reached in which it does not pay for either player to deviate from some action taken, given the action of the other. The payoff of players is going to be the steady state utility level of a representative generation. The game is a static one, and any dynamic interpretation to the "reaction functions" or best response correspondences of each player must be taken cautiously.

We divide the analysis in three games, according to the choice of regimes that we allow in each strategy space of the players.

In the first game, called the monetary-restricted game, we allow only the monetary regime to prevail in a world with perfect currency substitution. Each government must choose from a set of parameters compatible with a Competitive Monetary Equilibrium (MEF), given the actions of the other player. The range of parameters for which a worldwide inflation is compatible with a monetary equilibrium is going to be determined by the environment (preferences, initial endowments) and by the combined action of both players. Therefore, it could well be the case that, given the action of one of the players, the range of parameters from which the government may choose is such that it may end up in a lower payoff than before having access to the game (the utility associated with the original endowment vector, for instance). This makes this game not very interesting in itself. However, it proves to be a useful input in posterior games, as well as in throwing insights on the best responses, in the monetary range, of each player depending upon their relative size, endowment structure, etc. The optimality of the outcome of this game is studied as well.

In the second game, called the <u>unrestricted</u> game, the choice of the <u>non-monetary</u> regime becomes available. This is a regime, as indicated in Section II, where monies are not valued, yet the private agents of different countries can engage in barter trade. The choice of the non-monetary regime can be modelled in two ways for each of the players: in one fashion, the change of regime can be made at any stage of the game. In the other, the players solve first the monetary-restricted game, then compare their outcome with the one they could get by the choice of the alternative regime and decide. In this case, it can be shown that, both are strategically equivalent. In this game we analyze under which circumstances is the Nash equilibrium going to be one regime or the other, as well as the optimality of the outcome. We also investigate the scope for <u>international coordination</u>, that is, whether there exist jointly made choices of parameters that can make both players better off than the outcome of the non-cooperative game.

In the third game, called the <u>expanded</u> game, the choice of the <u>portfolio autarky</u> regime as defined in Section II, becomes available. We analyze under which circumstances does the available choice become the option of the players. We also study the optimality of the allocation associated with this outcome, and finally, we discuss the scope for <u>international coordination</u>, when countries can use capital controls as a way to isolate themselves from the inefficiencies transmitted through the fixed exchange rates regime.

In what follows I will consider the simplifying assumptions:

(i) $N^h = 1$, $h = i,j$ (one individual per generation in each country)

(ii) $U(c_{1h}, c_{2h}) = c_{1h} \cdot c_{2h}$, $h=i,j$

(iii) $G_h(t) = 0$ all t, $h=i,j$

The latter assumption will restrict the government choice to only one parameter, since from (2) the budget constraint of the government will become:

$$(8) \qquad \frac{M_h(t) - M_h(t-1)}{P_h(t)} = -\beta_h(t) w_{1h}(t)$$

Then, by choosing a $\beta_h > 0$, the government chooses a level of taxes <u>and</u> withdrawal of money from circulation (deflation) and by choosing $\beta_h > 0$ the government chooses to inflate and give transfers.

Once each government chooses a policy regime and a policy parameter, they allow the competitive mechanism to operate: each private agent solves his/her optimization problem depending upon the regime, as described in Section II, and some equilibrium is achieved. The steady state utility level associated with this equilibrium will be the payoff for each government.

For the specific case of the Stationary Competitive Monetary Equilibrium with perfect currency substitution, there are two ways to parameterize the policy choice:

(i) To choose a <u>sequence</u> of <u>nominal</u> money creation, $M^h(t) = \mu^h(t)M^h(t-1)$. Given the market structure, this determines the $1+r(t) = 1+r$ sequence. Given the initial conditions, it pins down the sequences $P_i(t)$ $P_j(t)$, so that the players can compute the <u>real</u> transfers/taxes to their residents and determine their steady state utility level.

(ii) To choose a single parameter, β_h, of <u>real</u> money creation, $-\beta_h w_i h$, such that, the market clearing condition, which embodies agents' optimization, determines $1+r$, and given that the tax/transfer scheme is already in real terms,

it follows that the steady state utility level can be computed.

In (27) I show the equivalence of both ways to support a Stationary Competitive Monetary Equilibrium. The <u>stationary</u> condition is crucial for such an equivalence: it implies a constant rate of inflation, and that each player will choose a constant rate of <u>real</u> money growth, so that there is a single choice of β_h, instead of a sequence $\beta_h(t)$. Given the equivalence, it will be more convenient to use the second approach throughout the exposition.

IV.2. THE NASH EQUILIBRIA AMONG THE MONETARY EQUILIBRIA: THE MONETARY-RESTRICTED GAME

Both countries are forced to choose a policy rule taht is consistent with the maintenance of the monetary regime, given the action of the other, no matter whether the outcome is worse than the utility associated with the non-monetary regime, or even the utility associated with their initial endowment vector.

Before formalizing the game, let's analyze the conditions under which a monetary equilibrium exists.

Lemma 1: Let $w_{1i} + w_{1j} > w_{2i} + w_{2j}$

A necessary and sufficient condition for a market equilibrium to exist is given by

$$(9)\quad -(w_{1i}\beta_i + w_{1j}\beta_j) = \delta \leq \emptyset_1 + 3\emptyset_2 - 2(2\emptyset_2^2 + 2\emptyset_1\emptyset_2)^{1/2} \equiv MAXSEIG \equiv \hat{\delta}$$

with strict inequality if $w_{2i} = w_{2j} = 0$, where $\emptyset_h = w_{hi} + w_{hj}$.

We will call (9) the "maximum seignorage condition", and it states the upper bound for goods that may be raised through fiat money creation, being consistent with the existence of a monetary regime.

When $w_{2h} = 0$, all h, the maximum seignorage condition must hold with strict inequality, $\delta < \emptyset_1$, in order for $(1+r) > 0$. The interpretation is clear: the governments cannot raise, in

aggregate, more goods than the ones available in the economy and still maintain a monetary equilibrium.

Given the monetary choices (β_i, β_j) from each country, the combined effect on the worldwide rate of return (inverse of rate of inflation) is given by:

$$(10)\quad 1+r = \frac{\emptyset_1 + \emptyset_2 - \delta + ((\emptyset_1 - \emptyset_2 + 3\delta)^2 - 8\delta(\delta + \emptyset_1))^{1/2}}{2(\emptyset_1 + \delta)}$$

The reader can verify this expression from reading the proof of Lemma 1 in the Appendix.

The Game: There are two players (governments) engaged in a non-cooperative game defined by

(a) strategy space for government i, $S_i(\beta_j)$

Define $S = \left\{(\beta_i, \beta_j) : 1+r(\beta_i, \beta_j) > \frac{\emptyset_2}{\emptyset_1 + \delta} \text{ and } \beta_j \leq 1\right.$

S is the set of possible pairs of policy parameters (β_i, β_j) for which a stationary monetary equilibrium (MEF) exists. In Figure 5 we present an exampleof how S looks like.

Then, $S_i(\beta_j) = \{\beta_i : (\beta_i, \beta_j) \in S\}$ is the strategy space for player i, given the actions of j.

In Game Theory literature (see, for example (20) or (22) it is not standard that the strategy space depends on the actions of the rival. Reformulating our strategy space in a way consistent with the standard convention would generate problems of continuity of the payoff function (for the pairs that break the monetary equilibrium). We do not use any existing results from that body of literature, and we will find convenient to limit ourselves to the above strategy space.

(b) Payoff function: $U_i(\beta_i,\beta_j)$ for player i.

Under a MEF, with those preferences

$$c_{1i} = \frac{1}{2}\left(w_{1i}(1-\beta_i) + \frac{w_{2i}}{(1+r)}\right) \text{ and } c_{2i} = c_{1i}(1+r).$$

$$\text{Define } U_i(\beta_i,\beta_j) = c_{1i}(\beta_i,\beta_j) \cdot c_{2i}(\beta_i,\beta_j)$$

$$= \frac{1}{4}\left(w_{1i}(1-\beta_i) + \frac{w_{2i}}{(1+r)}\right)^2(1+r), \text{ where}$$

$1+r(\beta_i,\beta_j)$ solves (10), to be payoff function for i when j chooses β_j and we restrict ourselves to $(\beta_i,\beta_j) \in S$.

Analogously define $U_j(\beta_i,\beta_j)$.

Define $R_i(\beta_j) = \arg\max_{\beta_i \in S_i(\beta_j)} U_i(\beta_i,\beta_j)$ to be the best response of player i, give β_j. Analogously define $R_j(\beta_i)$.

Definition 5: A Nash equilibrium for the restricted-monetary game (NE-M) is a pair $(\widetilde{\beta}_i,\widetilde{\beta}_j) \in S$ such that $\widetilde{\beta}_i \in R_i(\widetilde{\beta}_j)$ and $\widetilde{\beta}_j \in R_j(\widetilde{\beta}_i)$.

Proposition 1: Let $w_{1i} > 0$, all i. Let $w_{1i}+w_{1j} > w_{2i}+w_{2j}$. Then there exists a Nash equilibrium for the restricted monetary game. Moreover the NE is unique: $R_h(\beta_k) = \beta_h$ all h,k.

The proof is in the Appendix. The Existence proof follows a fixed point type of argument. It follows Theorem 3 in Debreu (5). The main difficulty was to prove quasiconcavity of the payoff function, in order for the best response correspondences to be well-defined. The payoff function is not concave everywhere. However, it is shown that it is strictly concave where the FONC are satisfied, and because of being differentiable, quasiconcavity follows. Another difficulty with the proof was that for $(\beta_i,\beta_j) = (1,1)$ there is no monetary equilibrium. There S is not closed, and the payoff functions are not continuous. A transformation of the set and the payoff functions had to be used in order to apply the above Theorem.

Proposition 2: Let $w_{1i}+w_{1j} > w_{2i}+w_{2j}$

Then, there is no Nash Equilibrium with zero inflation.

The idea is that if one country plays constant monetary policy, the other will always have an incentive to inflate. Moreover, if one country responds with deflation to an expansionary monetary policy of its rival, so that the final effect is absence of distortion (zero inflation), there sill be an incentive to the expansionary country to further monetize.

This Proposition is important since it implies that the Nash Equilibrium of the non-cooperative monetary game can be dominated.

<u>Corollary 1</u>: as $(\theta_1-\theta_2) \rightarrow 0$ the monetary Nash solution tends to a zero inflation equilibrium, which is also the <u>rate of return</u> of the non-monetary equilibrium when the aggregate first period endowments equal the second period. However, in the limit, the resulting allocation is the non-monetary equilibrium allocation only if $w_{1i} = w_{2i}$, $w_{1j} = w_{2j}$ (and $w_{1i}+w_{1j} = w_{2i} + w_{2j}$).

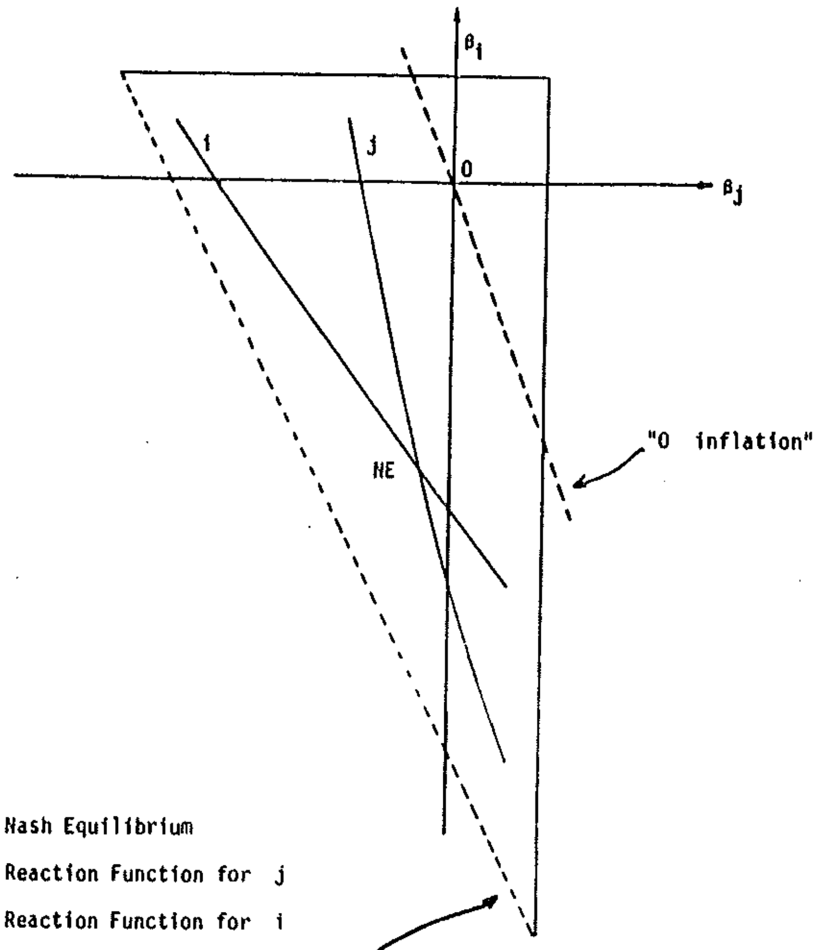
The interpretation is the following. As the world desire for savings diminishes, there is less room for the monetary game to be played, therefore a smaller inflation will be achieved under the Nash solution. In the limit, a zero inflation NE is the outcome. Such an outcome is the non-monetary solution if both countries have no desire for saving, that is $(\tilde{\beta}_i, \tilde{\beta}_j) \rightarrow (0,0)$ only if $w_{1i} = w_{2i}$ and $w_{1j} = w_{2j}$. If the countries' endowment differ, the NE will be a zero inflation, $\delta = 0$, but $\tilde{\beta}_i < 0$, $\tilde{\beta}_j > 0$ or viceversa. We shall see later that the country of borrowers will end up all through this game in a better position, and that will be the case for the limit situation as well.

Proposition 2 indicated that the NE will never be of a zero inflation (and therefore deflation) type. That restricts the location of the NE to the band limited between the boundary and the 0-inflation line of S. (See Figure 4).

We can now state the suboptimality of the outcome of

<u>Figure 4</u>

Strategy Space for the Case $(w_{1i},0)(w_{1j},0)$
Reaction Functions and Nash Equilibrium



NE: Nash Equilibrium

j: Reaction Function for j

i: Reaction Function for i

$w_{1j} = 3w_{1i}$

$-(w_{1i}\hat{\beta}_i + w_{1j}\hat{\beta}_j) = (w_{1i}+w_{1j}) = \phi_1 = \delta$

this game. This is interesting in order to analyze the scope for International Coordination.

In general, in a non-cooperative game, the Nash outcome is not Pareto Optimal, reproducing the Prisioner's Dilemma paradigm. This will be also the case for the game studied here. Howvever, as we shall see in a later section, this will not necessarily be the case when we allow for a richer strategy space for each player.

<u>Proposition 3</u>: The monetary-restricted NE is not Pareto Optimal.

<u>Proof</u>: Since $s_h(\ ) > 0$, some h, take without loss generality $s_i(\ ) > 0$. Together with $1+r < 1$ we can always find a

feasible allocation, preferred to this one, by taking within each country, ϵ_i from c_{1i}, giving ϵ_i to old for all $t \geq 1$.

We have characterized the main results concerning the outcome of this game: existence and uniqueness, inflationary solution, suboptimal allocation.

Other features regarding the NE of this game is that the <u>"maximum seignorage"</u> condition is never exhausted (that is the restricted monetary NE is always in the interior of the strategy space) and that in equilibrium the savings of both countries are equal. This happens no matter the size or the <u>"desire for savings"</u> (defined as the ratio between first and second period endowments). This can give us an idea of how can we expect a player to end up with the game depending upon its initial position. For a country of <u>"borrowers"</u>, $w_{1j} < w_{2j}$, it can be shown that it will end up getting goods from the other country (that is, $\tilde{\beta}_i > 0$, $\tilde{\beta}_j < 0$).

Then if a country is made of borrowers it will have a better position in the game. The country of lenders, in order to preserve the highest return on assets for their citizens will deflate (taxing and reducing their own money supply, lowering the worldwide rate of inflation) while the country of borrowers keeps inflating, getting goods from its rival (since their extra pieces of paper are still exchangeable for goods in the world market) and lowering the rate of return on savings.

The game is, therefore, a "game of borrowers", in the sense of being them the ones who take advantage of the potential gains associated with the fixed exchange rate and perfect currency substitution institutional arrangement. Even if countries are lenders, the one who is relatively more of it, that is, the one that has a "higher desire for saving" will end up in a lower after-tax period endowment, that is will be a loser of the game.

It is also a "game of small versus big". The reason is that the smaller the country, the less "noticeable" effect of their expansionary monetary policies in the worldwide rate of inflation, actually

$$\frac{\partial r/\partial \beta_j}{\partial r/\partial \beta_i} = \frac{w_{1i}}{w_{1j}}$$

(Again for formal proof of these features we refer to (27)).

Therefore, the smaller the country, the more likely that it will inflate largely in order to get goods, while the relatively bigger must play the opposite.

IV.3. THE NASH EQUILIBRIA AMONG MONETARY AND NON-MONETARY EQUILIBRIA: THE UNRESTRICTED GAME

We will now allow the non-monetary regime to be a strategy choice for either player. Before defining the game we show that for the non-monetary regime there is a pure parameter strategy, that is if a government chooses the non-monetary regime, then the optimal policy is to do nothing. (Recall that here the government can only use "fiscal policy", that is $\beta_h \geq 0$, since money has no value).

Lemma 2: Let $G_h(t) = 0$ for both $h=i,j$. Then $\beta_h = 0$, is the optimal parameter policy choice under the non-monetary regime, for $h=i,j$.

The Game: Given that there is a unique optimal policy under the non-monetary regime, independent of each other's actions, we can now define the game as follows:

(a) Strategy space for government i:

$$\bar{S}_i = S_i(\bar{\beta}_j) \cup \{\alpha_i^{NM}\}$$

where $S_i(\bar{\beta}_j)$ is the strategy space for the monetary-restricted game, and α_i^{NM} is arbitrary in the sense that it could take the form of any β_i that "breaks the system", that is, a rate of money creation that is not consistent with a stationary competitive monetary equilibrium.

Finally, note that $S_i(\beta_j)$ depends on the actions of the rival, while α_i does not, i.e., we can always choose a β_i that is inconsistent with a monetary equilibrium no matter the choice of the other country.

(b) <u>Pay-off function for government i</u>: the steady state utility level of country i's residents achieved through the policy taken from $\bar{S}_i$, either the utility associated with MEP (if the outcome is to stay in the monetary regime) or the utility of the non-monetary allocation. Formally,

$$U_i(\beta_i,\beta_j) = \begin{cases} U_i(\beta_i,\beta_j) & \text{if } \beta_i \in S_i(\beta_j),\ \beta_j \neq \alpha_j^{NM} \\ U_i^{NM}(.), & \text{if } \beta_i = \alpha_i^{NM},\ \beta_j = \alpha^{jNM} \text{ or both.} \end{cases}$$

The latter simply states that once the monetary regime is broken, it is so for <u>both</u> countries.

<u>Definition 6:</u> A <u>Nash Equilibrium</u> from the unrestricted game will be a pair $(\beta_i^*, \beta_j^*) \in S \cup \{(\alpha_i^{NM},\alpha_j^{NM})\}$ s.t.

$$U_i(\beta_i^*,\beta_j^*) \geq U_i(\beta_i,\beta_j^*), \quad \text{for all } \beta_i \in \bar{S}_i$$
$$U_j(\beta_i^*,\beta_j^*) \geq U_j(\beta_i^*,\beta_j), \quad \text{for all } \beta_j \in \bar{S}_j$$

As we mentioned before, we can break this game into a strategically equivalent two-step game which will be easier to deal with analytically.

Define g^* the game where the non-monetary option is available at each stage. Define g the monetary restricted

game and g^{**} the game in two steps: the restricted monetary, g, whose NE given by $(\tilde{\beta}_i,\tilde{\beta}_j)$ exists and is unique, as shown in Proposition 1, and an unrestricted game g^{**}, in strategic form where each country's strategy space is simply a pair of strategies (play <u>"monetary"</u> or <u>"non-monetary"</u>) and whose payoff table is given by

	Monetary: $\tilde{\beta}_j$	Non-Monetary: α_j^{NM}
Monetary: $\tilde{\beta}_i$	$U_i(\tilde{\beta}_i,\tilde{\beta}_j)$, $U_j(\tilde{\beta}_i,\tilde{\beta}_j)$	U_i^{NM}, U_j^{NM}
Non-Monetary: α_i^{NM}	U_i^{NM}, U_j^{NM}	U_i^{NM}, U_j^{NM}

where $U_h(\tilde{\beta}_i,\tilde{\beta}_j)$ is the NE of g, where $U_i^{NM} = U(c_{1i},c_{2i},1+r_{NM})$ and where $1+r_{NM}$ solves $s_i(1+r_{NM}) + s_j(1+r_{NM}) = 0$ for $s_h = w_1^h - c_1^h$, h=i,j. As mentioned earlier, in (27) is shown that both games are strategically equivalent, that is $NE(g^*) \leftrightarrow NE(g^{**})$.

The following says that at least one of the countries, prefer the Nash-monetary solution to the non-monetary equilibrium:

<u>Lemma 3</u>: $U_h(\tilde{\beta}_i,\tilde{\beta}_j) > U_h^{NM}$ for at least one h.

We should be a little careful in interpreting the choice (β_k,α_h^{NM}). The govenment k chooses a sequence $\{M_k(t)\}$ which, for a constant inflation and constant real money growth corresponds to a unique β_k, when $1/P_k(t) > 0$ all t. The choice (β_k, α_k^{NM}) corresponds to government k choosing such a nominal sequence, yet $1/P(t)$ becoming $= 0$, and the non-monetary regime prevailing. Here the assumption of printing money being costless happens to be crucial, since otherwise a lower payoff would have to be assigned to such choices, and the strategical equivalence of both games as well as the process for obtaining Perfect Equilibrium from eliminating dominated strategies would not hold.

<u>Perfect Equilibria</u>

Given the equivalence between g^{**} and g^{*} we now focus on the former game specificacion.

We can characterize NE(g^{**}) in the following way. Let, without loss of generality $U_i(\tilde{\beta}_i, \tilde{\beta}_j) > U_i^{NM}$. Then

$$
\begin{aligned}
NE(g^{**}):\quad & (\alpha_i^{NM}, \alpha_j^{NM}), (\tilde{\beta}_i, \tilde{\beta}_j) && \text{if } U_j(\tilde{\beta}_i, \tilde{\beta}_j) > U_j^{NM} \\
& (\alpha_i^{NM}, \alpha_j^{NM}), (\tilde{\beta}_i, \alpha_j^{NM}) && \text{if } U_j(\tilde{\beta}_i, \tilde{\beta}_j) < U_j^{NM} \\
& (\alpha_i^{NM}, \alpha_j^{NM}), (\tilde{\beta}_i, \tilde{\beta}_j), (\tilde{\beta}_i, \alpha_j^{NM}) && \\
& && \text{if } U_j(\tilde{\beta}_i, \tilde{\beta}_j) = U_j^{NM}
\end{aligned}
$$

As we have seen, there are <u>multiple NE</u> in g^{**}. We will find interesting, as it is usually done in Game Theory literature, to study refinements of the Nash solution.

One way to do this is to focus on <u>Perfectness</u>, that is the equilibria reached by way of elimination of dominated strategies.

Here the process of elimination of dominated strategies is well defined because the entry in the last column and row of the payoff matrix of g^{**} correspond to a <u>same</u> value for each player: the steady state utility of the non-monetary regime. It's easy to build payoff tables where this would not be true. This is the reason why it is important to emphasize that the choice ($\tilde{\beta}_k, \alpha_k^{NM}$) was equivalent to (α_k^{NM}, α_k^{NM}), as mentioned above.

We now study Perfect Equilibria, (PE), in g^{**}.

<u>Proposition 4</u>: There is a unique <u>Perfect</u> Equilibrium in g^{**}, unless $U_h(\tilde{\beta}_i, \tilde{\beta}_j) = U_h^{NM}$ for some h.

<u>Proof</u>: From Lemma 3, $U_k(\tilde{\beta}_i, \tilde{\beta}_j) > U_k^{NM}$ for some k. Then β_k is a <u>dominant</u> strategy for player <u>k</u>. Let $U_h(\tilde{\beta}_h, \tilde{\beta}_k) > U_h^{NM}$ for $h \neq k$, then $(\tilde{\beta}_h, \tilde{\beta}_k)$ is the unique Perfect Equilibrium. let $U_h(\tilde{\beta}_h, \tilde{\beta}_k) < U_h^{NM}$, for $h \neq k$. Then (α_h^{NM}, β_k) is the unique Perfect Equilibrium.

<u>Non-monetary versus monetary NE</u>

Previously we characterized the restricted-monetary NE. Now we compare the outcome of such a game with the non-monetary allocation. We do so in order to analyze under which conditions there will be switch, due to one player, to the non-monetary regime.

We first summarize the main features relating to both equilibria. Recall our assumptions $\emptyset_1 > \emptyset_2 \geq 0$, $w_{ih} > 0$, both h.

Non-monetary equilibrium

(i) $1+r^{NM} = \frac{\emptyset_2}{\emptyset_1}$

(ii) $s_i(\ )^{NM} + s_j(\ )^{NM} = 0$ (market clearing condition)

(iii) $s_i(\ )^{NM} = \frac{1}{2}\left(w_{1i} - w_{2i} \frac{\emptyset_1}{\emptyset_2}\right)$

(iv) $U_i^{NM}(\ ) = (w_{1i} - s_i^{NM})*(1+r^{NM})$

Monetary-Nash

(v) $1+r = \frac{\emptyset_1+\emptyset_2-\delta+V}{2(\emptyset_1+\delta)} > \frac{\emptyset_2}{\emptyset_1}, \quad \delta > 0$

(vi) $s_i(\ ) = s_j(\ ) > 0$

(vii) $s_i(\ ) = \frac{1}{2}\left(w_{1i}\ (1-\tilde{\beta}_i) - \frac{w_{2i}}{(1+\tilde{r})}\right)$

(viii) $(s_i(\ ) + s_j(\ ))(-\tilde{r}) = \delta$

(ix) $U_i(\ ) = (w_{1i}(1-\tilde{\beta}_i) - s_i(\ ))*(1+\tilde{r})$

Note that if $w_{2i} = w_{2j} = 0$. Then, the Perfect Equilibrium of the unrestricted game will always be the monetary outcome (β_i,β_j), since (iii) and (iv), $w_{2i} = 0$ yields

$$U_i^{NM}(\ ) = U_j^{NM}(\ ) = 0$$

Lemma 4: If both countries are identical, the Perfect Equilibrium of the unrestricted game will always be the monetary outcome $(\tilde{\beta}_i,\tilde{\beta}_j)$. If one country is made of borrowers, $w_{2i} > w_{1i}$, and (*) $(1-\tilde{\beta}_i)*(1-\tilde{r}) < \frac{\emptyset_2}{\emptyset_1}$ then the Perfect

Equilibrium of the unrestricted game will always be non-mone

tary. (Condition (*) refers to countries' endowments not being too disparate).

Proposition 5: The Perfect Equilibrium of the unrestricted game is not Pareto optimal.

Proof: If the outcome is Nash-monetary, see Proposition 3. If the outcome is non-monetary, $1+r^{NM} = \frac{\emptyset_2}{\emptyset_1} < 1$. Therefore an identical argument applies.

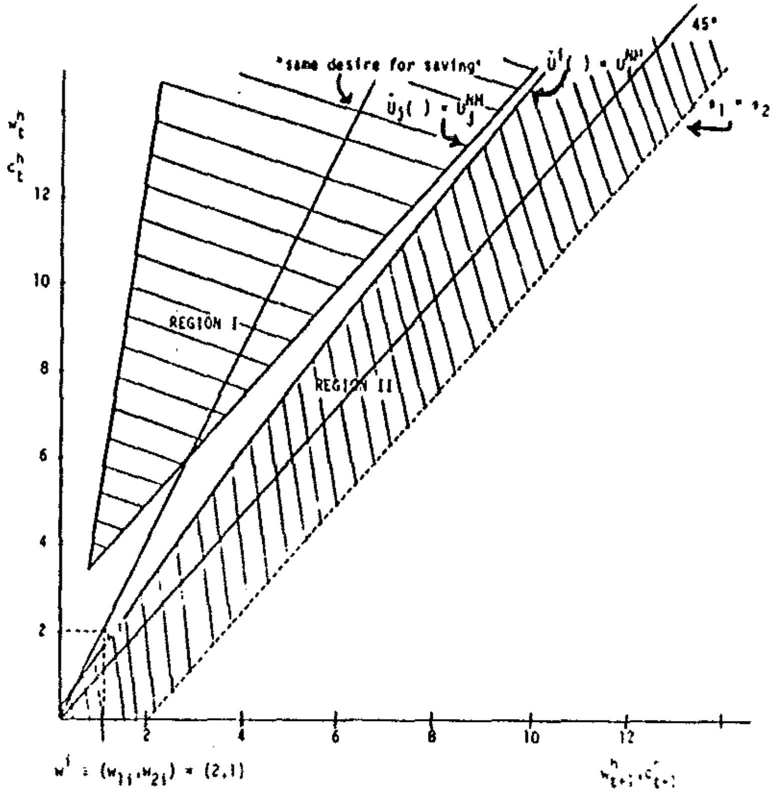
As we stated above, sometimes the PE ends up being the monetary regime, sometimes the non-monetary. There is a borderline, one for each player, in which it can be either. However, for most of the environments, the non-monetary regime is most likely to prevail.

I now present a numerical example. Let's fix, arbitrarily, the endowments of country i, $w^i \equiv (w_{1i},w_{2i})$. Without loss of generality let $w_{1i} > w_{2i}$. Then let's study the outcome of the monetary regime and compare it to the non-monetary allocation. We will analyze, then, given w^i, which w^j will make the game end up in a non-monetary regime (either because i or j decided to switch) or the monetary one (because both of them preferred the monetary-Nash solution).

We can define three regions regarding the possible outcomes depicted above (see Figure 5).

Figure 5

Non-Monetary Versus Monetary-Nash



(i) Region I: in which $U_j(\widetilde{\beta}_i,\widetilde{\beta}_j) < U_j^{NM}$, that is, country j switches to the non-monetary regime.

(ii) Region II: in which $U_i(\widetilde{\beta}_i,\widetilde{\beta}_j) < U_i^{NM}$, therefore country i switching to the non-monetary regime.

(iii) Region III: in which $U_h(\widetilde{\beta}_i,\widetilde{\beta}_j) > U_h^{NM}$, both h (i.e., both prefer the monetary outcome).

Notice that Regions I and II do not overlap; that is, there is always at least one country preferring the monetary outcome. (This is trivial in a world, where in aggregate, first period endowment are larger than second periods).

In Figure 5 the three regions are graphed, for the $w^i \equiv$ (2,1). Region III is just the unshaded area.

Region I: Country j switching to the non-monetary regime. It occurs when country j is relatively bigger than i and with relatively same desire for saving. The bigger the country j, the more transfers to country i, the more likely to switch. The ability of country i to tax j depends on j's desire for savings. Therefore, if j and i are similar in endowments size, but j has a stronger desire for saving, then the more powerful will i be in taxing away their endowments, therefore the more likely j is going to switch.

Region II: depicts the conditions under which i prefers the non-monetary regime. In general if j is a net borrower

$(w_{1j} < w_{2j})$, i will switch to the non-monetary regime. This seems counterintuitive at first glance: why will a country of lenders prefer a non-monetary regime which provides a lower return, while the borrowers would rather stick to the higher interest rate monetary regime? The explanation is in the complicated payoffs regarding the ability of getting transfers (or being taxed), and the welfare loss associated with inflation, for each country. The borrower has relatively more dislike from the monetary regime versus the non-monetary because of the higher interest rates. Yet, they are to be the big winners from exploiting the fixed-exchange rate monetary game for two reasons: on one hand, they are able to get higher real transfers from the country of lenders, which will more likely match any inflation with domestic taxation ($\beta_h > 0$) in order to keep their savings' return. On the other hand, a higher rate of inflation means lower return for their loans for the borrowers. Still they will not exploit their advantage to the limit (that is, the <u>"maximum seignorage condition"</u> is not binding under the NE) because throughout the monetary game described above the game <u>borrowers</u> will end up being after-transfers <u>lenders</u> (since $s_i(\ ) = s_j(\ ) > 0$).

<u>Region III</u>: In general, the <u>monetary</u> regime will <u>prevail</u> in the following three cases:

(a) If both countries are very <u>similar</u>, both in endowments, size and in desire for savings pattern, that is, in a neighborhood of w'. (This is an extension of Lemma 3 if both countries are identical the monetary regime prevails).

(b) If the country which is relatively more of <u>lenders</u> is also relatively <u>larger</u> in first period endowments, i.e., when the big country has also a strong desire for saving relative to the smaller one. (The case $(w_{1i},0)(w_{1j},0)$ will be included in this case).

(c) If both countries have basically the same <u>difference</u> in endowments, but the bigger the country the more desire for saving (that is, a "45' plus" line through w_1). This is the area that separates Regions I and II, and gets smaller as one country becomes larger from the other.

<u>The scope for international coordination for the unrestricted game.</u>

We have shown that the outcome of the unrestricted game is not P.O.

We now consider the case where both governments, instead of choosing policies in a non-cooperative way, make a joint decision that may improve their payoff (welfare level of their citizens) associated with the non-cooperative solution. The scope for that cooperation behavior is studied, that is, the existence of joint strategies that yield a better outcome than the Nash solution for both countires.

It should be noted here that as it is usual in the literature concerning ways to achieve Pareto Superior outcomes to the Nash solution, there is a question of how to implement these cooperation schemes. The problem is <u>enforceability</u>, since there is always an incentive to deviate from the agreement. One way to get around it is to write a contract in which players might be penalized in deviating. A more sophisticated way to get around the enforceability problem is to generate a <u>Repeated</u> game,(see (22) for example) whose non-cooperative solution is the coordinated solution of the static game. We will not focus on the implementation problem, but rather in showing the <u>existence</u> of coordination schemes no matter the outcome of the unrestricted game. We will also focus on characterizing how should that coordinated pair of policies look like depending upon the endowment structure.

<u>Definition 7:</u> A <u>coordination scheme</u> is a pair $(\hat{\beta}_i,\hat{\beta}_j)$, jointly decided by both players, such that,

(i) $(\hat{\beta}_i,\hat{\beta}_j) \in S$, the resulting allocation and prices $(c, P_h, 1+r)$ being a competitive monetary equilibrium (MEF).

(ii) $U_h(\hat{\beta}_i,\hat{\beta}_j) \geq U_h(\beta_i^*,\beta_j^*)$ for both $h=i,j$, with strict inequality for at least some h, and where (β_i^*,β_j^*) represents the Nash equilibrium of the unrestricted game (see Definition 6).

As before, we divide the analysis in two parts, one regarding the scope for coordination when the outcome of the non-cooperative game is the monetary one, the other when a non-monetary regime prevails.

<u>Case 1: When the NE is the Non-Monetary</u>

For that case $U_h^{NM} = \frac{1}{4}\left(w_{1h}+w_{2h}\frac{\emptyset_1}{\emptyset_2}\right)^2 \frac{\emptyset_2}{\emptyset_1}$; $h=i$.

A coordination scheme $(\hat{\beta}_i,\hat{\beta}_j)$ will have to fulfill (i)

$(\hat{\beta}_i,\hat{\beta}_j) \in S$, that is (ia) $\hat{\beta}_h \leq 1$, all h, with strict inequality for some h. (ib) $w_{1i}\hat{\beta}_i + w_{1j}\hat{\beta}_j \leq$ MAXSEIG $(\emptyset_1,\emptyset_2)$. (Recall that the case $w_{2i} = w_{2j} = 0$ forced to impose strict inequality, but this will not be the case here since the outcome of the Nash game will always be monetary when $w_{2i} = w_{2j} = 0$) and (ii) $\left(w_{1h}(1-\beta_h) = \frac{w_{2h}}{(1+r)}\right)^2 (1+r) \geq \left(w_{1h} + w_{2h}\frac{\emptyset_1}{\emptyset_2}\right)^2 \frac{\emptyset_2}{\emptyset_1}$, with strict inequality for some h.

<u>Lemma 5</u>: Let (11) $\left(\frac{\omega_{1h}}{\omega_{2h}}\right)^* > \frac{\theta_1}{\theta_2} > 1$ for all h. Then $(\hat{\beta}_i, \hat{\beta}_j) = (0,0)$ is a coordination scheme, when the Perfect Nash outcome is non-monetary.

The economic interpretation of the above Lemma is the following. When countries' endowments pattern satisfy (11), it means that they have a similar <u>"desire for saving"</u> structure.

In general $\frac{\omega_{1i}}{\omega_{2i}} > \frac{\theta_1}{\theta_2} > \frac{\omega_{1j}}{\omega_{2j}}$, if the desire for saving is similar, $\frac{\omega_{1i}}{\omega_{2i}}$ will be close to $\frac{\omega_{1j}}{\omega_{2j}}$ and so to $\frac{\theta_1}{\theta_2} > 1$. Then $\left(\frac{\omega_{1h}}{\omega_{2h}}\right)^* > \frac{\theta_1}{\theta_2}$ will hold for both countries. Thon <u>"do nothing"</u> will leave them as good as before playing that game with the suboptimal result. If there is disparity between countries' endowment distribution, the country which is relatively more of a borrower will more likely benefit from the lower market interest rate associated with the non-monetary regime. In such a case a rearrangement of the world equilibrium that leaves each country on their own, may make them worse off, unless some transfer payment is sent. In that case (0,0) cannot beat the non-monetary regime.

It can be shown that beacuse of aggregate endowments in the first period being larger than for the second, condition (11) must hold for at least one of the countries. The following states that when the condition fails to hold for one of the countries, we can still prove existence of a coordination scheme.

<u>Lemma 6</u>: Let $\left(\frac{\omega_{1j}}{\omega_{2j}}\right)^* < \frac{\theta_1}{\theta_2}$. Let the Nash solution be non-monetary.

Let $(\hat{\beta}_i, \hat{\beta}_j)$ be a MEF such that $\begin{cases} U_j(\hat{\beta}_i, \hat{\beta}_j) = U_j^{NN} & \text{(a1)} \\ 1+\hat{r} = 1 & \text{(b1)} \end{cases}$

and let $(\hat{\hat{\beta}}_i, \hat{\hat{\beta}}_j)$ be a MEF such that $\begin{cases} U_i(\hat{\hat{\beta}}_i, \hat{\hat{\beta}}_j) = U_i^{NN} & \text{(a2)} \\ 1+\hat{\hat{r}} = 1 & \text{(b2)} \end{cases}$

Then $\hat{\hat{\beta}}_j < \hat{\beta}_j < 0$, $0 < \hat{\beta}_i < \hat{\hat{\beta}}_i$ and $(\hat{\beta}_i, \hat{\beta}_j)$, $(\hat{\hat{\beta}}_i, \hat{\hat{\beta}}_j)$ and any $(\beta_i(\mu), \beta_j(\mu))$ where $\beta_h(\mu) = \mu\beta_h + (1-\mu)\beta_h$, $0<\mu<1$ such that $1+r(\mu) = 1$ are <u>coordination schemes</u>.

Therefore we can define

$C = \left\{ (\beta_i^c, \beta_j^c) : \hat{\beta}_i \le \beta_i^c \le \hat{\hat{\beta}}_i,\ \hat{\hat{\beta}}_j \le \beta_j^c \le \hat{\beta}_j \text{ and } \omega_{1i}\beta_i^c + \omega_{1j}\beta_j^c = 0 \right\}$ is the set of <u>0 inflation</u> coordination schemes when the outcome of the non-cooperative game is non-monetary. Note that there exists a continuum of inflationary and deflationary coordination schemes as well.

<u>Proposition 6</u>: The coordination schemes embodied in C are P.O.

<u>Proof</u>: All the conditions of Balasko-Shell theorem (1) apply, therefore $1+r = 1$ imply optimality of the resulting allocations.

Case 2: When the NE is Monetary

Call $(U_h(\tilde{\beta}_i,\tilde{\beta}_j)$ the utility levels associated with the NE$(\tilde{\beta}_i,\tilde{\beta}_j)$ for country h.

Clearly $\tilde{c}_{1i} + \tilde{c}_{2i} + \tilde{c}_{1j} + \tilde{c}_{2j} = w_{1i} + w_{2i} + w_{1j} + w_{2j}$, since it is a competitive monetary equilibrium.

Now take the allocation $(\check{c}_{1i},\check{c}_{2i})(\check{c}_{1j},\check{c}_{2j})$ such that $\tilde{U}_h = \check{U}_h$, all h and $\check{c}_{1h} = \check{c}_{2h}$ for all h (this would be equivalent to an equilibrium with $1+r = 1$). This allocation doesn't fulfill the market clearing condition, that is,

$$\check{c}_{1i} + \check{c}_{2i} + \check{c}_{1j} + \check{c}_{2j} < w_{1i} + w_{2i} + w_{1j} + w_{2j}$$

This was argued in Proposition 4 when we showed the non-optimality of the monetary Nash equilibrium. It was shown that there is an allocation that is P.S. to it, just by taking the difference between the aggregate endowments and the

allocation $(\check{c}_{1i}, \check{c}_{2i})$ $(\check{c}_{1j}, \check{c}_{2j})$ and distributing it in any fashion among the residentes of either country.

Although the allocation $(\check{c}_{1i},\check{c}_{2i},\check{c}_{1j},\check{c}_{2j})$ cannot be supported through a coordinated choice (β_i,β_j) since it is not a MEF, it provides room for existence of a coordination scheme as stated in Definition 7.

Lemma 8: Let the outcome of the Nash game be monetary, $(\tilde{\beta}_i,\tilde{\beta}_j)$.

Let $(\hat{\beta}_i,\hat{\beta}_j)$ be a MEF such that $\hat{U}_i(\ ) = \check{U}^i(\ ) = \tilde{U}^i(\ )$ (a3)
$1+\hat{r} = 1$ (b3)
Let $(\hat{\hat{\beta}}_i,\hat{\hat{\beta}}_j)$ be a MEF such that $\hat{\hat{U}}_j(\ ) = \check{U}^j(\ ) = \tilde{U}_j(\ )$ (a4)
$1+\hat{\hat{r}} = 1$ (b4)

Then $(\hat{\beta}_i,\hat{\beta}_j)$, $(\hat{\hat{\beta}}_i,\hat{\hat{\beta}}_j)$ and any $(\beta_i(\mu),\beta_j(\mu)$ where $\beta_h(\mu) = \mu\hat{\beta}_h + (1-\mu)\hat{\hat{\beta}}_h$, $0 < \mu < 1$, such that $1+r(\mu) = 1$ are coordination schemes.

Proposition 7: A coordination scheme always exists. The Proof follows directly from the previous three Lemmas.

Note that, in general, countries agreeing not to do anything is not a coordination scheme. However it can be shown that, under some cases, that will be the case. In particular, if both countries are identical, (0,0), is a coordination scheme. In Figure 6 we present other environments for which (0,0) is a coordination scheme.

It can also be shown that if one of the countries is made of borrowers, then an optimal (zero inflation) coordination scheme must include some "transfer payments" from the country of lenders to the country of borrowers, that is the country of borrowers inflating, the country of lenders deflating.

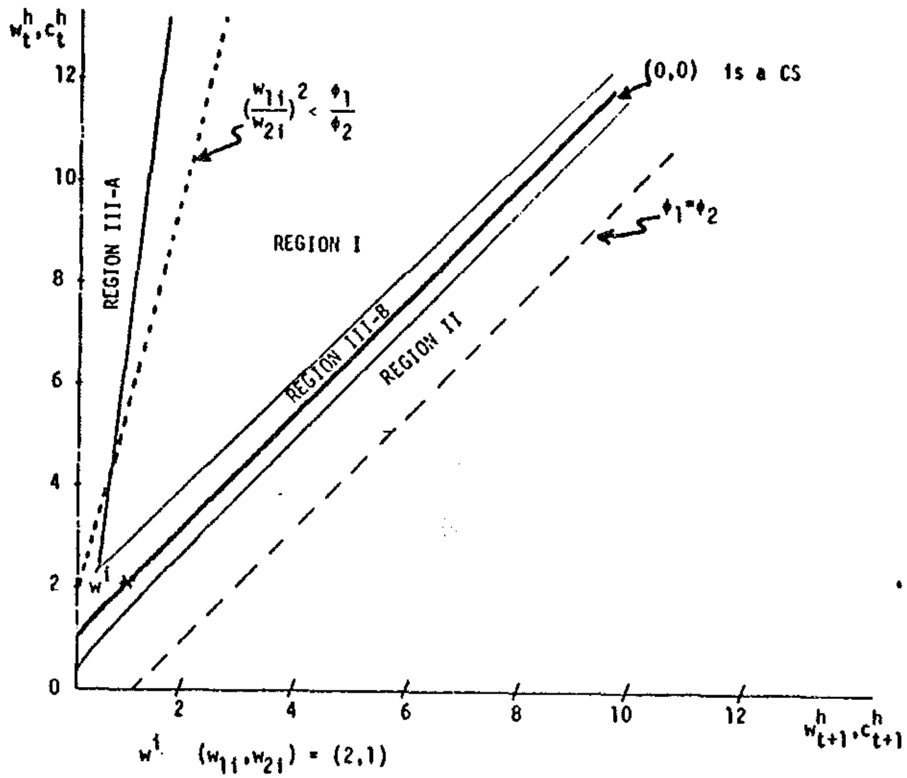
In Figure 6, we divide Region III, the region for which a monetary-Nash is the outcome in two subregions:

Region III-A, in which the endowment pattern (desire for saving) is similar and where the monetary-Nash falls in between both "non-monetary regime" areas. In the middle of such a region there will be an area such that (0,0) is preferred by both to the Nash outcome. (The (0,0) area is so small that it looks like a line, but it actually has some positive measure). Further from that area some transfers are needed to maintain a zero inflation world equilibrium. The more we move to the left (country j more of a lender) the more transfers required for i (the winner of the monetary-Nash) and vice versa.

Region III-B: At the left of the non-monetary region for player j, in which player i takes even more advantage of him/her, and therefore would have to have more transfers than such a coordination scheme that beats the non-monetary outcome. Therefore, it is not surprising that we cannot find (0,0) as a coordination scheme in such an area.

Figure 6

An Example of Environments for Which (0,0) is a Coordination Scheme



IV.4. THE NASH EQUILIBRIA AMONG THE MONETARY, NON-MONETARY AND PORTFOLIO AUTARKY EQUILIBRIA: THE EXPANDED GAME.

In this section we allow for countries to use a richer set of choices in their non-cooperative game.

Before, they were deciding between switching to a non-monetary regime, in which governments lose their ability to affect the welfare of their citizens, or maintaining the monetary regime without capital controls, and where some equilibrium level of monetary choices was reached.

Now we allow them to impose some restrictions on their holdings of foreign assets and on the private international borrowing and lending market (capital controls)

The way we model this restriction is by allowing each country to "go on their own", that is, to prevent their citizens from holding foreign currency as a store of value, and to not allow private loans between members of different countries.

Both restrictions, when imposed jointly, relax the condition of equal equilibrium rates of return for both monies. There can be dominance in rate of return, yet each country may hold their own currency. Let us assume that only the first restriction is imposed, for each country, and yet private international borrowing and lending is permitted. Then, both monies return have to be at least as big as the private loans rate, and strictly equal if such a market matters at all. Therefore there would not be any difference between imposing that restriction and the laissez-faire regime studied above.

As we showed in Section II, the imposition of both restrictions, called "portfolio autarky" (PA), in which both countries go on their own, allow for different equilibrium rates of return in each country, and for different welfare levels than those of the monetary and non-monetary Nash solutions studied previously.

We first show that there is a pure strategy for this regime, that is, if a government chooses the portfolio autarky regime, then the optimal policy parameter is to do nothing.

<u>Optimal government choice under PA</u>: Each government's objective will be the steady state utility level, that is choose σ_i to

$$\max U_i(\sigma_i) = \left(w_{1i}(1-\sigma_i) + \frac{w_{2i}}{(1+r_i)}\right)^2 (1+r_i)$$

where the agents' optimization required

$$(7) \qquad \frac{P_h(t+1)}{P_h(t)} = \frac{1}{1+r_h(t)} \text{ for each } h, \text{ all } t.$$

The market clearing condition associated with a MEPA, and (7) yield

$$\tfrac{1}{2}\left(w_{1i}(1-\sigma_i) - \frac{w_{2i}}{(1+r_i)}\right)(-r_i) = -\sigma_i w_{1i}$$

or (12)

$$1+r_i = \frac{w_i(1+\sigma_i) + w_{2i} + \left((w_{1i}(1-3\sigma_i) - w_{2i})^2 + 8w^2_{1i}\sigma_i(1-\sigma_i)\right)^{1/2}}{2w_{1i}(1-\sigma_i)}$$

Any σ_i consistent with (7), money being valued, must also verify the 1-country version of the <u>"maximun seignorage"</u> condition, that is

$$(13) \qquad 1 + r_i(\sigma_i) \geq \frac{w_{2i}}{w_{1i}}$$

where $r_i(\sigma_i)$ follows (12).

Each government will choose σ_i in order to maximize the steady state utility level subject to (12) and (13). The FONC are:

$$\left(w_{1i}(1-\sigma_i) - \frac{w_{2i}}{(1+r_i)}\right) r'_i = 2(1+r_i)w_{1i}, \quad \text{where } r'_i = \frac{\partial r(\sigma_i)}{\partial \sigma_i}$$

<u>Remark</u>: $\sigma_i = 0$ is the optimal choice for monetary policy under a PA regime.

<u>Proof</u>: Let $\sigma_i = 0$. Then $(1+r_i)=1$. But for a MEPA, $w_{1i} \geq w_{2i}$, therefore (13) holds.

Also $a_i(\quad) = (w_{1i}-w_{2i})$ and it can be shown that $r'_i = \frac{2w_{1i}}{w_{1i}-w_{2i}}$.

Then $a_i(.)r'_i = 2w_{1i}$, so the FONCs are satisfied.

From the single-valuedness of $U_i(\sigma_i)$, $\sigma_i = 0$ is the unique solution.

<u>Non-Monetary Equlibrium Under PA</u>: From the previous Remark the payoff function associated with the PA strategy choice becomes

$$U_i^{PA} = \tfrac{1}{4}\,(w_{1i} + w_{2i})^2$$

when there is room for a MEPA, that is when $1+r_i = 1 \geq \frac{w_{2i}}{w_{1i}}$.

When $\frac{w_{2i}}{w_{1i}} > 1$, there is no MEPA, but we can still define a <u>non-monetary</u> equilibrium, in which $1 + r_i = \frac{w_{2i}}{w_{1i}}$ and U_i^{PA} $(\ ) = w_{1i}\ .\ w_{2i}$. Since there is homogeneity, the welfare level associated with the non-monetary equilibrium of the closed economy is such that the consumption allocation equals the endowment allocation (no room for trade across generations or between countries).

Before we formally define the game we should mention that once the portfolio restrictions are imposed in one country, they have to be imposed in the other country, that is, there cannot be asymmetric catpital controls.

<u>Assymmetric Capital Controls</u>: In (27) I show that if one country plays PA (that is, it doesn't allow its residents to hold foreing money) the other country must also impose restrictions on holding foreing currency, or else have their savings being taxed away. Such a situation happens when $w_{1h} > w_{2h}$ for both h. If $w_{1h} < w_{2h}$, one-sided restrictions are

meaningless, since country h would be condemned to the non-monetary regime, and closure of international borrowing and lending will force them to their autarky-endowment allocation, independently of whether the restrictions are <u>"one-sided"</u>.

<u>The Expanded Game</u>: We can now define the expanded game, with the portfolio autarky choice as follows:

(a) <u>Strategy space</u> for player i, $\bar{\bar{S}}_i$

$$\bar{\bar{S}}_i = S_i(\beta_j) \cup \left\{ \alpha_i^{NM} \right\} \cup \left\{ \sigma_i^{PA} \right\}$$

where $S_i(\beta_j)$ is the strategy space for the monetary restricted game, α_i^{NM} represents switching to the non-monetary regime (and "do nothing") and σ_i^{PA} represents switching to the portfolio autarky regime (and "do nothing").

(b) <u>Payoff for each player</u> are represented in the following payoff matrix:

	Monetary	Non- monetary	P. Autarky
Monetary	$U_i(\tilde{\beta}_i,\tilde{\beta}_j), U_j(\tilde{\beta}_i,\tilde{\beta}_j)$	$U_i^{NM}, U^j NM$	U_i^{PA}, U_j^{PA}
Non-monetary	U_i^{NM}, U_j^{NM}	U_i^{NM}, U_j^{NM}	$w_{1i}\,.\,w_{2i}, U_j^{NM}$
P. Autarky	U_i^{PA}, U_j^{PA}	$U_i^{PA}, w_{1j}\,.\,w_{2j}$	U_i^{PA}, U_j^{PA}

The new cells added when PA option is introduced require some explanation.

Clearly, if both countries play PA, the resulting outcome will be (U_i^{PA}, U_j^{PA}). If one country plays PA and the other plays monetary, the latter will achieve the PA welfare (independently of whether or not a domestic monetary equilibrium can hold) since the country is left on their own, in terms of private loans, and because of what is argued above, it hau to also impose currency insubstitutability (if available).

If one country plays PA that shuts down the private loans market, the other country playing "non-monetary" is just achieving the utility associated with their endowments.

We will show that, contrary to the unrestricted game, here the pair (non-monetary, non-monetary), called $(\alpha_i^{NM}, \alpha_j^{NM})$ will never be a NE. In the unrestricted game it was always the case.

We first show some remarks characterizing the payoff associated with different strategies.

Then we proceed to show that in some cases there is a unique Nash Equilibrium associated with this game, which happes to be the Portfolio Autarky regime. In any other case, the payoffs associated with the Nash Equilibria will still be the ones of the Portfolio Autarky regime.

Finally, we study the optimality of such an outcome and the scope for International Coordination when the autarkic regime becomes available.

Given our assumption $\emptyset_1 > \emptyset_2$, let, from now on, $\frac{w_{1i}}{w_{2i}} > 1$, without loss of generality.

Then, it follows immediately that country i will never play non-monetary as best response to j's plying PA, since ¼ $(w_{1i} + w_{2i})^2 > w_{1i} - w_{2i}$, and $U_i^{NM} \geq w_{1i}\, w_{2i}$, with strict inequality if countries are not identical, since

$$\tfrac{1}{4}\left(w_{1i} + w_{2i}\frac{\emptyset_1}{\emptyset_2}\right)^2 \frac{\emptyset_2}{\emptyset_1} \geq w_{1i} \cdot w_{2i} \cdot \left(\text{If } \frac{w_{2i}}{w_{1i}} = \frac{\emptyset_2}{\emptyset_1},\right.$$

equality holds).

Before analyzing the possible NE of the expanded game with PA, we now compare different payoff values under different environments.

Let $w_{1j} < w_{2j}$. Then $U_j(\widetilde{\beta}_i, \widetilde{\beta}_j) > U_j^{NM} > U_j^{PA}$, since

$$\left(w_{1j} + w_{2j}\frac{\emptyset_1}{\emptyset_2}\right)^2 \frac{\emptyset_2}{\emptyset_1} > w_{1j} w_{2j} = U_j^{PA}$$

We also showed that if $w_{1j} < w_{2j}$, $U(\widetilde{\beta}_i, \widetilde{\beta}_j) > U_j^{NM}$, the borrower country would alwasy be better off under the restricted monetary game.

<u>Remark</u>: Let $w_{1i} > w_{2i}$. Then the payoff associated with the PA choices is the same as the (0,0) coordination scheme payoff. Since, if $w_{1i} > w_{2i}$, there exists a MEPA and

$U_i^{PA} = \tfrac{1}{4}(w_{1i} + w_{2i})^2 = U_i(0,0)$.

<u>Definition 8</u>: Given (w_{1i}, w_{2i}) define

$$TR_i(w_{1i}, w_{2i}) = \left\{(w_{1j}, w_{2j}) : U_i(\widetilde{\beta}_i, \widetilde{\beta}_j) > U_i(0,0)\right\}$$

that is TR_i is the region, in the endowment space of $(\omega_{1j},\omega_{2j})$ such that "country i needs a transfer", that is, a (0,0) coordination scheme is not possible since country i is better off under monetary-Nash.

In Figure 6, this is the area on the <u>left</u> of the (0,0) coordination scheme area.

In that region, since $\frac{\omega_{1i}}{\omega_{2i}} > 1$, $U_i^{PA} > U_i^{NM}$. Then $U_i(\tilde{\beta}_i,\tilde{\beta}_j) > U_i(0,0) = U_i^{PA} > U_i^{NM}$.

<u>Remark</u>: If $(\omega_{1j}^*,\omega_{2j}^*) \in TR_i(\omega_{1i}^*,\omega_{2i}^*)$ then $(\omega_{1i}^*,\omega_{2i}^*) \notin TR_j(\omega_{1j}^*,\omega_{2j}^*)$ and vice versa. (The Proof follows from Propositions 2 and 4).

<u>Lemma 9:</u>

Let $(\omega_{1j},\omega_{2j}) \in TR_i$.

Then (PA,PA)(PA,NM) are the NE of the expanded game. The NE is (PA,PA) if $\omega_{1j} > \omega_{2j}$.

Moreover let $(\omega_{1j},\omega_{2j})(\omega_{1i},\omega_{2i})$ be such that a (0,0) coordination scheme is possible.

Then (PA,PA) is the NE of the expanded game.

Finally, let $(\omega_{1j},\omega_{2j}) \notin TR_i$ and (0,0) not be a coordination scheme. Then (PA,PA)(PA,NM) are the NE of the expanded game. The NE is (PA,PA) if $\omega_{1j} > \omega_{2j}$).

<u>Proposition 8</u>: (PA,PA)(PA,NM) is the NE of the expanded game. Moreover it is unique, (PA,PA), if $\omega_{1j} > \omega_{2j}$.

The <u>proof</u> follows directly from Lemma 9. The basic idea is that it will always be the case that one of the countries chooses PA, the other not being able to do better except with PA, unless $\omega_{1j} \leq \omega_{2j}$ in which case it is indifferent between PA or NM.

<u>Corollary 3</u>: In the cases where the NE of the expanded is not unique, $\omega_{1j} \leq \omega_{2j}$, the payoffs are still the ones of the Portfolio Autarky regime.

<u>Proposition 9</u>: Let $\omega_{1j} < \omega_{2j}$. Then the NE of the expanded game is not Pareto optimal. Let $\omega_{1j} > \omega_{2j}$. Then the NE of the expanded game is Pareto optimal.

<u>Proof</u>: For the first part, from Proposition 8, NE will be (PA,PA), (PA,NM).

Under PA, if $\omega_{1j} < \omega_{2j}$, $U_j^{PA} = \omega_{1j}\omega_{2j}$ and, since a MEPA, cannot exist,

$$1 + r_j = \frac{\omega_{2j}}{\omega_{1j}} > 1.$$

But $1+r_i = 1$, hence equality of Marginal Rates of Substitution across countries for all dates is violated.

For the second part, from Proposition 8, (PA,PA) is the unique NE. Then $1 + r_h = 1$ both h. Given $\omega_{1h} > \omega_{2h}$ for both, Balasko-Shell applies.

The scope for international coordination when PA is available.

Can countries now do better by getting together and choosing some joint $(\hat{\beta}_i, \hat{\beta}_j)$ rather than being on their own?

First let $w_{1i} > w_{2i}$, $w_{1j} > w_{2j}$. Then, there is no scope for international coordination. It follows from the Portfolio Autarky (NE of the Expanded game) being Pareto Optimal.

On the other hand, let $w_{1j} < w_{2j}$ for some j. Then there is scope for international coordination.

To prove it, note that we can find policy choices implying goods being transferred from the borrower country, j, which is doing badly under PA, in order for i to keep "open" the international borrowing and lending market.

In particular, any $(\hat{\beta}_i, \hat{\beta}_j)$ such that

(i) $1 + \hat{r} = 1$

(ii) $\hat{\beta}_j > 0$ such that $\frac{1}{4}\,(w_{1j}(1-\hat{\beta}_j) + w_{2j})^2 > w_{1j} w_{2j} = U_j^{PA}$

(iii) $\hat{\beta}_i < 0$ such that $\frac{1}{4}\,(w_{1i}(1-\hat{\beta}_i) + w_{2i})^2 > \frac{1}{4}\,(w_{1i} + w_{2i})^2 =$

$$= U_i^{PA}$$

will be a coordination scheme for the world with PA choice, and where country j "pays" $-w_{1i}\hat{\beta}_i = w_{1j}\hat{\beta}_j$ in order for i to open the private lending/borrowing market.

IV.5. SUMMARY OF RESULTS

In this Section we have studied the Nash solution of a non-cooperative game between two countries engaged in an inflation tax war over each other, given that the fixed exchange rate regime and the absence of portfolio restrictions give both monetary authorities access to the world money supply. The potential advantages of such policies may be partially or fully offset by the social welfare loss associated with the distortion (inflation).

The analysis is carried on in three stages, depending on whether the strategy spaces of each player include just the monetary-laissez faire regime with perfect currency substitution, or allow for the non-monetary regime to be adopted, or in the third stage, if the additional monetary-portfolio autarky regime option is introduced.

For the first stage, it is shown that when a country is relatively a lender and/or relatively bigger in their first period endowments, is the potential loser of the monetary game. The reason is that the smaller the country (or their taxable first period endowments) the less noticeable effect on the world inflation their monetary policies will be, and the less social cost inflation will have for them, because of their smaller desire for saving. On the other hand, the bigger a country, the more effect their policies on the world inflation rate, commonly suffered by both countries. Also, the more of a lender structure, the more concern about the final return on savings, and therefore the more likely to give in by deflating, which means imposing taxes on own residents and giving transfers to the other country in order to get a higher worldwide rate of return. In this stage, we showed existence of a Nash equilibrium for the monetary game. It is also shown that the monetary-Nash equilibrium is inflationary, therefore suboptimal. And when countries are identical (or very similar) this situation reproduces a version of the Prisoners' Dilemma result.

Allowing the option of the <u>non-monetary</u> regime does not improve matters in terms of efficiency: in most of the cases, for endowments patterns that are not very close, there will be a switch by the relatively lender or by the relatively bigger, to the non-monetary regime. The environment was chosen so that the non-monetary allocation can always be dominated. The rate of return associated with the non-monetary regime will be lower than under any inflationary monetary equilibrium, thus being further away from optimal.

This leaves room for studying <u>joint</u> choices of policies that dominate the outcome of the Nash solution, that is, to analyze the scope for international <u>coordination</u>. The results are that, for countries with similar endowment pattern I mean that the ratio of first to second period endowments must be similar, and decreasing as the first period endowments increase so that effects, lender structure and size structure offset each other. The countries are "similar" in affecting (being affected by) the international seignorage-distortion

game. For the rest of the cases, the cooperation scheme must include policy parameters that imply a real transfer from one player to the other, the one who is relatively a winner under the non-cooperative game, in order for the coordinated choice to be accepted.

When the game with a <u>portfolio autarky</u> option is introduced, it will always be the case that at least one of the countries decides going on their own. Therefore, it turns out that portfolio autarky (foreing exchange controls and restrictions to international borrowing and lending) is a Nash equilibrium for all the possible endowments combination. Furthermore, it is the unique Nash equilibrium when both countries are lenders. This fits the observed phenomenon of countries imposing such restrictions in the history of fixed exchange rate regimes. It will also be the case that when such an option is introduced, the game switches from one of borrowers to one of lenders, the latter becoming the potential winners of the expanded game. The outcome shown to be Pareto optimal in the cases involving two countries that are lenders, no matter the relative size and the endowment pattern. For the cases in which one of the countries is a borrower, it will be willing to accept joint policy choices that imply some goods being transferred to the other country, yet the country of borrowers preferring this to the Portfolio Autarky regime. Therefore such a cooperative allocation dominates the Nash solution.

CONCLUSIONS

This paper presents a model in which in a world with fixed exchange rates and absence of capital controls, non-cooperative games yield multiple Nash equilibria with non-optimal allocations, when we introduce regimes (i.e., either the monetary or the non-monetary regime) as strategies for the players. In general there is a unique Perfect Equilibrium for the game with both strategies. The sub-optimality of the non-cooperative solution provides scope for international coordination, in the form of joint choice of non-inflationary policies that dominate the Nash equilibria allocations.

When the portfolio autarky option is introduced as a strategy choice, it becomes a Nash equilibrium of the game. When it is the unique one, it is optimal, therefore leaving no room for cooperation. The two-country perfect currency substitution framework can be reinterpreted as a single-economy with disperse power to create deficit, say in different institutions, or autonomous communities, that take into account the welfare of a differentiated subset of agents within an economy. This paper will thus show the need for coordination of deficit policies of different institutions within an economy.

New directions for research should include generalizations of the specific assumptions used throughout, particularly regarding stationarity and the specific Cobb-Douglas preferences. It should also include a dynamic version of this

static game and a discounted future stream of utilities substituting for the steady state utility as a payoff function. It should also take into account that different government spending paths than the one used here would require each player to face an optimal combination of monetary and fiscal policies, instead of the single parameter choice presented above.

Finally, in order to overturn the result above that portfolio autarky regime to be the outcome of the expanded game, when available in the strategy space, attention should focus on models that separate currency from other assets, so that portfolio autarky is not equivalent to zero trade balance, as it was here. In such environments portfolio autarky, modelled as complete isolation from each other, will imply a much higher economic cost for both countries.

FOOTNOTES

1. See Ingram's (12) of the events of the late 50's and 60's regarding the collapse of the Bretton Woods System.

2. Cooper, (3) and (4), has analyzed the issue of policy interdependence and the necessity of coordination in a broad analysis.

3. Helpman and Razin, (11) compare alternative exchange rate systems in frameworks where governments' monetary policies are passive.

4. Hamada (8) has used a strategic approach to analyze these questions in a non-optimizing framework and where the outcomes of different policies are not related explicitly to the well-being of people, but to general "desirable goals" such as "price stability" and "external balance".

5. In an equilibrium framework, in which countries' choices are modelled as a game, several papers have examined the question of macropolicy and coordination. Wallace and Miller (19) study coordination within a different monetary game, while Kehoe (15) and (16) examines a fiscal policy game.

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